TRINITY COLLEGE

MATHEMATICS APPLICATIONS

UNITS 3 and 4
Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

| Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	12	12	100	100	$66\frac{2}{3}$
Total				150	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8   **(4 marks)**
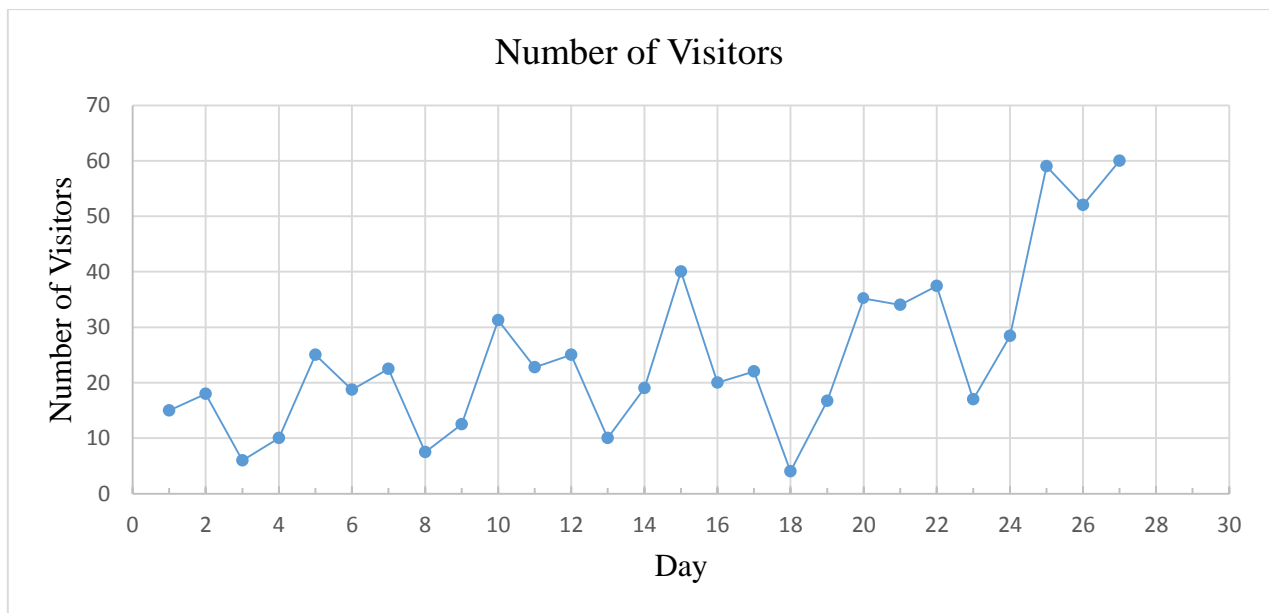
(a)   Suggest, with reasons, an appropriate moving average to smooth the given set of data.
(2 marks)

(b)   Describe long term trend in this time series.   (1 mark)

(c)   Describe any unusual fluctuations in this time series.   (1 mark)

Trinity College   **Applications Year 12**   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 9   **(8 marks)**

(a)   Describe a suitable method to organise and display data when investigating the existence of an association between two categorical variables.   **(2 marks)**

(b)   A class was set a task to investigate whether an association exists between the distance a student lived from school and the number of times they were late in a term.

    (i)   What **type** of graph would be appropriate to display data collected?   **(1 mark)**

    (ii)   What statistical measure would be useful to calculate in order to determine whether an association existed?   **(1 mark)**

    (iii)   One student designed the questionnaire shown below. Comment on the appropriateness of their design for this investigation.   **(2 marks)**

Name:

Tick one box	Distance less than 2 km	Distance more than 2 km
Late less than 3 times		
Late more than 3 times		

    (iv)   A student carried out the investigation, found that a moderate negative association existed, and concluded that frequent lateness was caused by living close to the school. Comment on their conclusion.   **(2 marks)**

Trinity College   **Applications Year 12**   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 10   **(7 marks)**

The number of votes still to count at the end of an election decreased by 72 every minute after 6 pm. At 6 pm, 2955 votes still needed counting.

(a)   Show that by 6:02 pm, 2811 votes still needed counting.   (1 mark)

(b)   Deduce a non-recursive rule for T_n, the number of votes still needing counting n minutes after 6 pm.   (2 marks)

(c)   Determine how many votes still needed counting at 6:30 pm.   (1 mark)

(d)   At 6:30 pm, counting slowed so that only 36 votes were processed every minute. Determine the time, to the nearest minute, that counting finished.   (3 marks)

Trinity College   **Applications Year 12**   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 11   **(17 marks)**

The table below shows the quarterly electricity bill for a 5 person household over a four year time period.

Time (t)	Year	Quarter	Bill ($)	4 pt. CMA	Bill as a % of Quarter
1		Q1	781		129.04%
2	2012	Q2	620		102.44%
3		Q3	450	612.13	74.35%
4		Q4	570	621.88	94.18%
5		Q1	836	625.63	130.32%
6	2013	Q2	643	634.00	a
7		Q3	457	b	71.24%
8		Q4	630	647.50	98.21%
9		Q1	837	663.88	123.63%
10	2014	Q2	689	675.75	101.77%
11		Q3	c	689.88	80.06%
12		Q4	640	696.63	94.53%
13		Q1	940	692.75	131.38%
14	2015	Q2	640	705.25	89.45%
15		Q3	560		78.27%
16		Q4	722		100.91%

(a)   Determine the values of *a, b* and *c* in the table.   (3 marks)

(b)   Fill in the two missing labels on the graph below.   (2 marks)

Electricity Bills
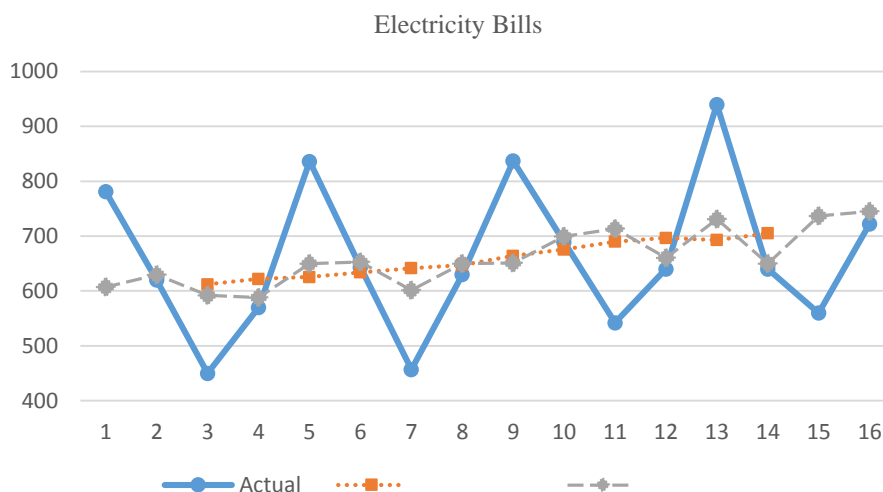
(c)   With reference to the graph, explain whether the 4 point centred moving average or deseasonalised data does a better job of smoothing the time series.   (2 marks)

(d)   Determine the seasonal quarterly index for Q1.   (2 marks)

(e)   Comment on the cost of the Q1 bills in relation the average quarterly bill.   (2 marks)

(f)   Describe any long term fluctuations (cycles) in this time series.   (1 marks)

(g)   Determine the equation of the regression line $M = A + Bt$.   (2 marks)

(h)   Use your regression line from (g) to predict the value of the Q1 bill in 2017.   (3 marks)

Trinity College   Applications Year 12   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 12   **(8 marks)**

A media company sought responses from the general public to the question "*How much trust do you have in the following for information about asylum seekers?"*. The company was investigating whether the source of information was associated with the degree of trust the general public placed in the information about asylum seekers.

The responses are summarised in the table below.

| | Degree of trust in asylum seeker information | | |
Information source	Some trust	Little trust	Not sure
Politicians	27	117	16
The media	25	84	11
Doctors	99	61	20
Churches	54	80	16

(a)   Name the explanatory and response variables for this investigation.   (2 marks)

(b)   Complete the table of percentages below, rounding to the nearest whole number, so that it can be used to identify whether the source of information is associated with the degree of trust the general public place in the information.   (4 marks)

| | Degree of trust in information | | |
Information source	Some trust	Little trust	Not sure
Politicians			
The media			
Doctors			
Churches			

(c)   Comment on whether this data provides any evidence that the source of information is associated with the degree of trust placed in the information about asylum seekers.
    (2 marks)

The daily customer satisfaction index was measured by an online business over a period of ten consecutive days and the data collected is shown in the table below.

Day (d)	1	2	3	4	5	6	7	8	9	10
CS Index (s)	92.1	91.2	90.6	88.9	88.1	87.7	87.4	86.6	85.4	85.1

(a)   Plot the missing data on the axes below.   (2 marks)
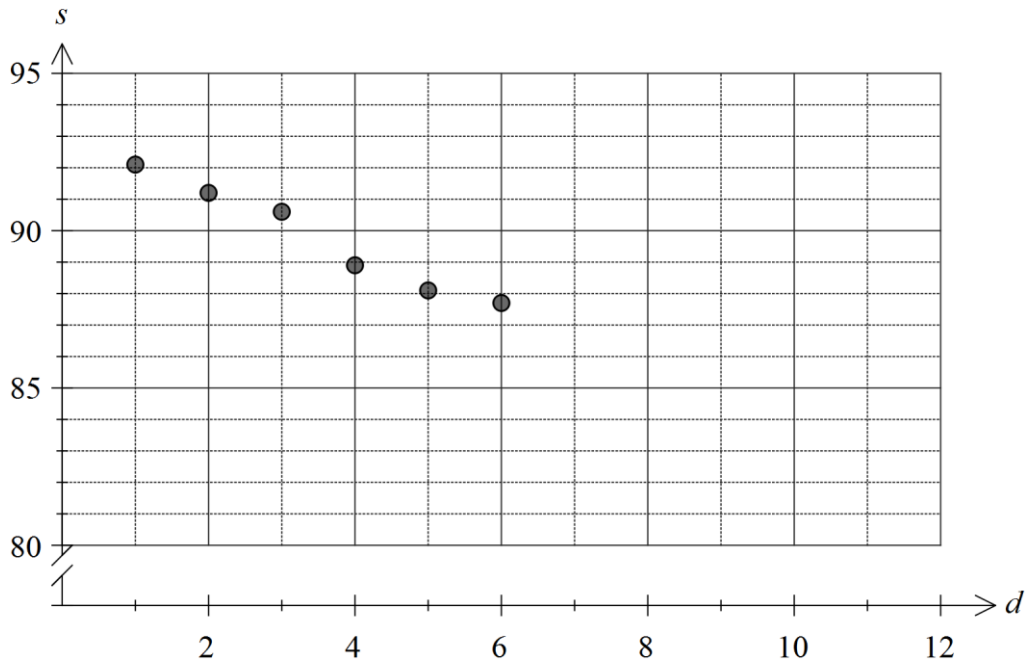
(b)   Determine the equation of the least-squares line that models the linear relationship between the day number and the customer satisfaction index.   (2 marks)

(c)   Draw the least-squares line on the axes above.   (2 marks)

(d)   Predict the customer satisfaction index for day 11.   (1 mark)

(e)   Explain why a prediction for the customer satisfaction index for day 15 should be treated with caution.   (1 mark)

See next page

Trinity College   **Applications Year 12**   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 14   **(9 marks)**

Sequence T is defined given by $T_{n+1} = 1.25T_n$, $T_1 = 50$.

(a) Use the recursive rule to complete the table below, rounding values to one decimal place.   (2 marks)

n	1	2	3	4	5	6
T_n	50					

(b) Graph the first six terms of sequence T on the axes below.   (2 marks)
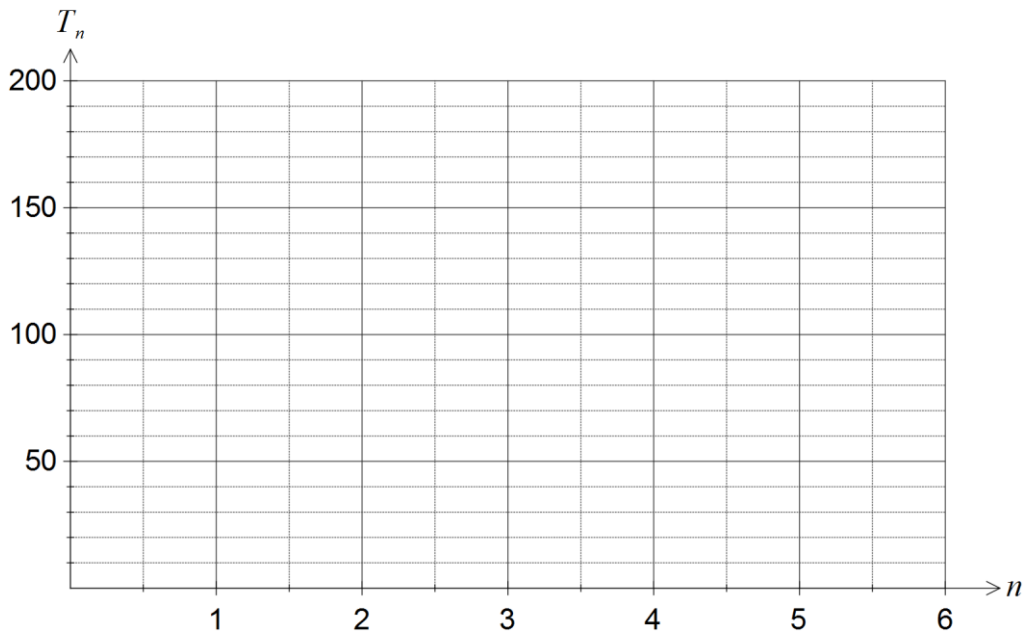
The first three terms of the geometric sequence U are 200, 160 and 128.

(c) Deduce a rule for the nth term of sequence U.   (2 marks)

(d)   Determine U_{10} .   (1 mark)

(e)   Determine the largest value of n so that $U_n > T_n$, justifying your answer.   (2 marks)

Trinity College   **Applications Year 12**   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 15   **(9 marks)**

A business has branches in six cities. The table below shows the time, in minutes, it takes for a package received at one branch to be transported to a branch in another city, where a direct route exists.

	P	Q	R	S	T
Q	40				
R	-	50			
S	-	25	30		
T	30	35	-	25	
U	25	-	-	65	35

(a)   Construct a weighted graph to show this information, using the cities placed below.

                                                                    (3 marks)

P

Q

R

T

S

U
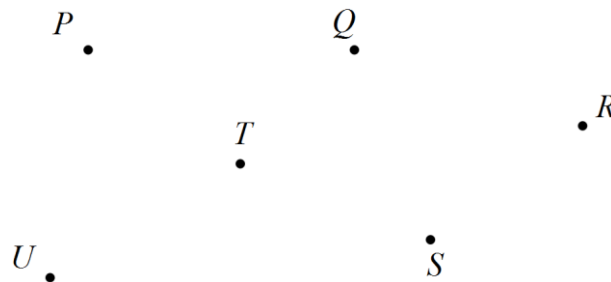
(b)   Determine the shortest transport time for a package to travel from

      (i)   P to S.   (1 mark)

      (ii)   Q to U.   (1 mark)

(c)   A document needs to be sent from branch U via branch R, where a customer will sign the document, to branch P. Determine the minimum transport time for the document to make this journey, listing all branches on the way.   (2 marks)

(d)   Another business document requires signing by the manager of each branch. In planning a route for this document, would finding a Eulerian trail be more appropriate than finding a Hamiltonian trail? Explain your answer.   (2 marks)

Question 16   **(7 marks)**

A fish farmer initially stocked a tank with 50 small fish. At the end of each month, the farmer caught some of the largest fish and sold them before adding more, smaller fish to the tank.

The number of fish in the tank at the start of the nth month is given by F_n, where
$$F_{n+1} = 0.7F_n + 120, \quad F_1 = 50.$$
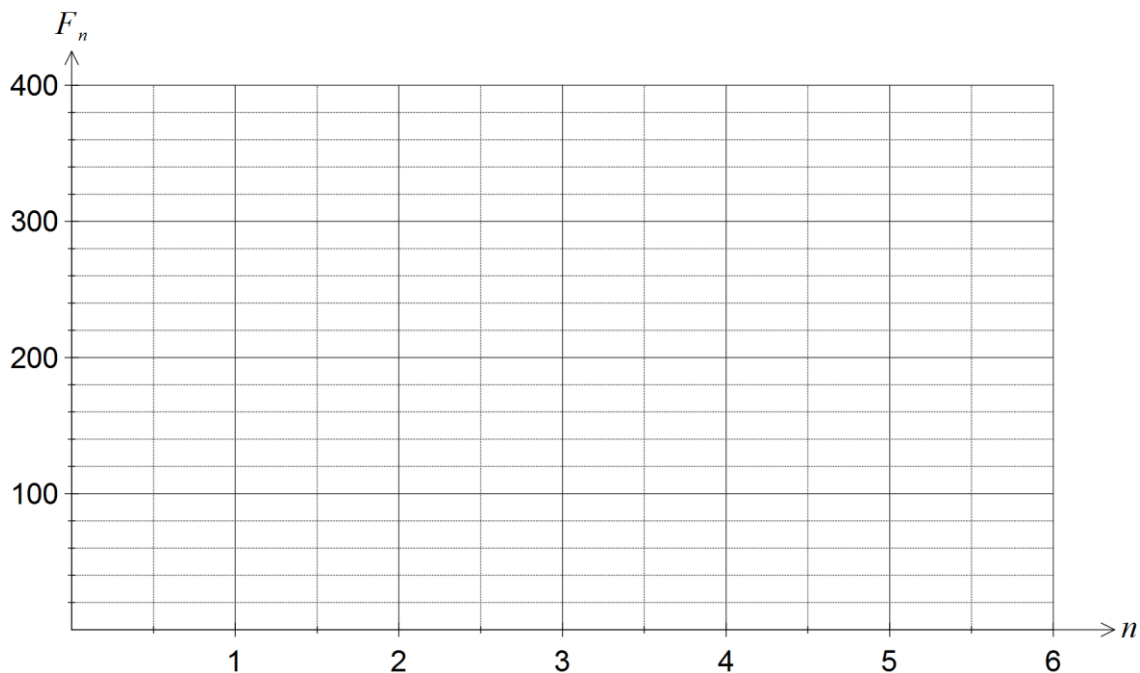
(a)   Use the recurrence relation to state

         (i)   the number of smaller fish added to the tank each month.   (1 mark)

         (ii)   the percentage of the fish caught and sold each month.   (1 mark)

(b)   Graph F_n on the axes below for $1 \le n \le 6$.   (3 marks)

(c)   Assuming this model continues, comment on how the number of fish in the tank changes over the next few years.   (2 marks)

Trinity College   **Applications Year 12**   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 17   **(9 marks)**

An airline has flights between six cities as shown in the graph below. Two of the flights are sightseeing flights that return to the city from which they departed.
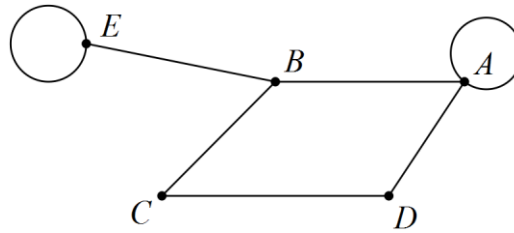
(a)   Determine M, the adjacency matrix for this graph.   (3 marks)

(b)   Calculate M^2 and explain the significance of the elements in this matrix that are zero.   (3 marks)

(c)   Determine the number of zero elements in the matrix $M + M^2$ and explain their significance in terms of specific flight(s).   (3 marks)

Trinity College   **Applications Year 12**   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 18   **(6 marks)**

From observations of a random sample of 236 blackbirds, the equation of the least-squares line that models the relationship between the wing span (s, measured in centimetres) and the mass (m, measured in grams) of blackbirds was found to be $s = 0.085m + 28.4$. The coefficient of determination between the variables was 0.79.

(a)   State the percentage of the variation in wing span of blackbirds that can be explained by the variation in their mass.   (1 mark)

(b)   Calculate the correlation coefficient between s and m, using the fact that the direction of the association is positive.   (1 mark)

(c)   Predict the wing span of a blackbird with a mass of 98 grams.   (1 mark)

(d)   Explain why it is difficult to comment on the reliability of the prediction in (c).   (2 marks)

(e)   The mean mass of the birds in the sample was 84.8 grams. Determine the mean wing span of birds in the sample.   (1 mark)

Trinity College   **Applications Year 12**   **Semester 1 2016**
Section 2   **Calculator Assumed**
Question 19   **(8 marks)**

An art gallery records the value of all artworks at the start of each year for insurance purposes. The first valuation of a picture was $4 800, and at the start of the next two years the picture was valued at $5 040 and $5 292 respectively.

(a)   Show that the picture values form a geometric sequence.   (2 marks)

(b)   Assuming that the value of the picture continues to increase in this way,

      (i)   calculate the increase in value of the picture during the third year.   (2 marks)

      (ii)   calculate the insurance premium for the picture in the tenth year, if the premium is 2.5% of the value of the picture.   (2 marks)

      (iii)   determine the year in which the insurance premium, still 2.5% of the value of the picture, will first exceed $300.   (2 marks)

END OF QUESTIONS

Trinity College
Section 2
Additional working space

Applications Year 12

Semester 1 2016
Calculator Assumed

Question number: _____

Trinity College   Applications Year 12   **Semester 1 2016**
Section 2   **Calculator Assumed**
Additional working space

Question number: _____